higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

T1050**(E)**(M30)T

NATIONAL CERTIFICATE

MATHEMATICS N5

(16030175)

30 March 2017 (X-Paper)
09:00–12:00

Scientific calculators may be used.

This question paper consists of 6 pages and a formula sheet of 5 pages.

DEPARTMENT OF HIGHER EDUCATION AND TRAINING
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE
MATHEMATICS N5
TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
2. Read ALL the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Show ALL intermediate steps and simplify where possible.
5. ALL final answers must be rounded off to THREE decimal places.
6. Questions may be answered in any order, but subsections of questions must be kept together.
7. Questions must be answered in blue or black ink.
8. Write neatly and legibly.

QUESTION 1

1.1 1.1.1 Determine $\ln y$ if: $\ln y = \lim_{x \to 9} \frac{\sqrt{x} - 3}{x - 9}$ (2)

1.1.2 Now, determine the value of y (1)

1.2 Determine $\lim_{x \to 1} \frac{(1-x)^2}{x^2 - x - x \ln x}$ (4)

[7]

QUESTION 2

2.1 Given: $y = \frac{1}{\sqrt{4}\, x^2}$

Determine the derivative $\frac{dy}{dx}$, using first principles. (4)

2.2 Determine $\frac{dy}{dx}$ in each of the following cases (simplification is NOT required):

2.2.1 $y = \arcsin(e^{\frac{1}{2}x} + 3)$ (2)

2.2.2 $y = \frac{3x}{\tan^3(\frac{4}{x} - 5)}$ (4)

2.2.3 $y = e^{\ln(\cos^2 x)}$ (2)

2.3 Determine $\frac{dy}{dx}$ with the aid of logarithmic differentiation if $y = \frac{\sqrt{x^2 - 1}.\tan 4x}{\ln x}$ (4)

2.4 Given: $y = \arctan x$

2.4.1 Make a neat, rough sketch of the graph of y for the range $[\frac{-\pi}{2}; \frac{\pi}{2}]$ (3)

2.4.2 Derive a formula to determine $\frac{dy}{dx}$ if $y = \arctan x$ (2)

2.5 Given: the implicit function $e^{\sec y} - \frac{1}{x} = 2y^2$

Calculate the derivative $\frac{dy}{dx}$. (4)

[25]

QUESTION 3

3.1 Given: $f(x) = x^3 + x^2 - 2x - 1$

3.1.1 Determine the coordinates of the point of inflection of $f(x)$. (2)

3.1.2 Draw up a table of values for x and $f(x)$ with x ranging from x = 1,1 to x = 1,4 using intervals of 0,1. Round answers off to ONE decimal.

Draw a neat graph of $f(x)$ between these values. (4)

3.1.3 An estimated value of the root of the equation $x^3 + x^2 - 2x - 1 = 0$ is $x = 1{,}23$.

Use Taylor/Newton's method to determine a better approximation of this root. (4)

3.2 An open cardboard box with a square base is required to hold 205 dm^3.

Determine the dimensions of the box if the area of the cardboard used should be as small as possible. (5)

3.3 An object moves in a straight line. After t seconds the distance x meters from a fixed point on the line is given by

$$x = \frac{1}{3}t^3 - t^2 - 8t - 1$$

Obtain an expression for the velocity of the object after t seconds and then calculate the values of t when the object is at rest. (4)

[19]

QUESTION 4

4.1 Determine $\int \frac{x-8}{x^2-2x-3}dx$ by resolving the integral into partial fractions. (5)

4.2 Determine $\int y dx$ in each of the following cases:

4.2.1 $\int \frac{\cot \sqrt[3]{x}}{\sqrt[3]{x^2}}dx$ (3)

4.2.2 $\int x^3 . 6^{-2x^4} \quad dx$ (3)

4.2.3 $\int \sin 3\pi x . \cos 8\pi x \quad dx$ (2)

4.2.4 $\int \frac{x^2}{3x-2}dx$ (5)

4.3 Given: $\int f(x).g'(x) = f(x).g(x) - \int f'(x).g(x)dx$

Determine $\int x^{7.} \ln x^5 dx$ (4)

[22]

QUESTION 5

5.1 Given: $y = e^x$ and $y = x^3$

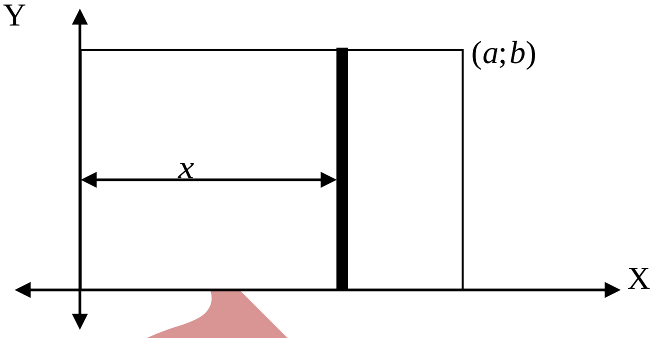
5.1.1 Make a neat sketch to show the area enclosed between the two curves, the Y-axis and the line $x = 1{,}5$. Also show the representative strip and the shaded area. (3)

5.1.2 Calculate the magnitude of the enclosed area. (4)

5.1.3 Calculate the volume generated when this area rotates about the X-axis. (4)

5.2 Calculate the value of $\int_0^1 \frac{1}{\sqrt{16-9x^2}}dx$ (4)

5.3 Determine the second moment of mass of a rectangular lamina of mass (m) about the axis parallel to one side of the lamina. The distance from the representative strip to the axis is x. The lamina's length and breadth is a and b respectively.



(4)
[19]

QUESTION 6

6.1 Determine the general solution of

$$\frac{dy}{dx} = \frac{2x^2 - 1}{10^{2y}}$$

(3)

6.2 Determine the general solution of the differential equation

$$4\frac{d^2y}{dx^2} = -2e^{3x} + \frac{1}{\cos^2 3x} - 4{,}8$$

(5)
[8]

TOTAL: 100

MATHEMATICS N5

FORMULA SHEET

Any applicable formula may also be used.

TRIGONOMETRY

$\sin^2 x + \cos^2 x = 1 \rightarrow \cos^2 x = 1 - \sin^2 x$

$\sin^2 x + \cos^2 x = 1 \rightarrow \sin^2 x = 1 - \cos^2 x$

$1 + \tan^2 x = \sec^2 x \longrightarrow \tan^2 x = \sec^2 x - 1$

$1 + \cot^2 x = \operatorname{cosec}^2 x \longrightarrow \cot^2 x = \operatorname{cosec}^2 x - 1$

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$\sin^2 A = ½ - ½ \cos 2A = ½(1 - \cos 2A)$

$\cos^2 A = ½ + ½ \cos 2A = ½ (1 + \cos 2A)$

$\sin (A \pm B) = \sin A \cos B \pm \sin B \cos A$

$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$

$$\tan (A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$\sin A \cos B = ½ [\sin (A + B) + \sin (A - B)]$

$\cos A \sin B = ½ [\sin (A + B) - \sin (A - B)]$

$\cos A \cos B = ½ [\cos (A + B) + \cos (A - B)]$

$\sin A \sin B = ½ [\cos (A - B) - \cos (A + B)]$

$$\tan x = \frac{\sin x}{\cos x} ; \sin x = \frac{1}{\operatorname{cosec} x} ; \cos x = \frac{1}{\sec x}$$

$$\cot x = \frac{\cos x}{\sin x} ; \operatorname{cosec} x = \frac{1}{\sin x} ; \sec x = \frac{1}{\cos x}$$

BINOMIAL THEOREM

$$(x+h)^n = x^n + nx^{n-1}\,h + \frac{n(n-1)}{2!}x^{n-2}\,h^2 + \frac{n(n-1)(n-2)}{1\times 2\times 3}x^{n-3}h^3 + \ldots$$

DIFFERENTIATION

$$e = -\frac{f(a)}{f^{'}(a)}$$

$$r = a + e$$

PRODUCT RULE

$$y \quad = u(x)\,.\,v(x)$$

$$\frac{dy}{dx} = u\,.\,\frac{dv}{dx} + v\,.\,\frac{du}{dx}$$

$$= u \cdot v^{'} + v \cdot u^{'}$$

QUOTIENT RULE

$$y \quad = \frac{u(x)}{v(x)}$$

$$\frac{dy}{dx} = \frac{v\,.\,\frac{du}{dx} - u\,.\,\frac{dv}{dx}}{v^2}$$

$$= \frac{v \cdot u' - u \cdot v'}{v^2}$$

CHAIN RULE

$$y \quad = f(u(x))$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$f(x)$	$\frac{d}{dx}\, f(x)$	$\int f(x)dx$
x^n	nx^{n-1}	$\frac{x^{n+1}}{n+1} + c \quad (n \neq -1)$
ax^n	$a\frac{d}{dx}x^n$	$a\int x^n dx$
e^{ax+b}	$e^{ax+b} \cdot \frac{d}{dx}(ax+b)$	$\frac{e^{ax+b}}{\frac{d}{dx}(ax+b)} + C$
a^{dx+e}	$a^{dx+e}.\ln a\, \frac{d}{dx}(dx+e)$	$\frac{a^{dx+e}}{\ln a . \frac{d}{dx}(dx+e)} + C$
$\ln(ax)$	$\frac{1}{ax} . \frac{d}{dx}ax$	$x\ln ax - x + C$
$e^{f(x)}$	$e^{f(x)} . \frac{d}{dx} f(x)$	—
$a^{f(x)}$	$a^{f(x)} .\ln a . \frac{d}{dx} f(x)$	—
$\ln f(x)$	$\frac{1}{f(x)} . \frac{d}{dx} f(x)$	—
$\sin ax$	$a \cos ax$	$-\frac{\cos ax}{a} . + C$
$\cos ax$	$-a \sin ax$	$\frac{\sin ax}{a} . + C$
$\tan ax$	$a \sec^2 ax$	$\frac{1}{a}\ln[\sec(ax)] + C$
$\cot ax$	$- a\, \mathrm{cosec}^2\, ax$	$\frac{1}{a}\ln[\sin(ax)] + C$
$\sec ax$	$a \sec ax \cdot \tan ax$	$\frac{1}{a}\ln[\sec ax + \tan ax] + C$
$\mathrm{cosec}\, ax$	$- a\, \mathrm{cosec}\, ax \cdot \cot ax$	$\frac{1}{a}\ln\left[\tan\left(\frac{ax}{2}\right)\right] + C$

$f(x)$	$\frac{d}{dx}f(x)$	$\int f(x)dx$
$\sin f(x)$	$\cos f(x) \bullet f^{1}(x)$	___
$\cos f(x)$	$-\sin f(x) \bullet f^{1}(x)$	___
$\tan f(x)$	$\sec^2 f(x) \bullet f^{1}(x)$	___
$\cot f(x)$	$-\ \mathrm{cosec}^2 f(x) \bullet f^{1}(x)$	___
$\sec f(x)$	$\sec f(x) \bullet \tan f(x) \bullet f^{1}(x)$	___
$\mathrm{cosec}\ f(x)$	$-\ \mathrm{cosec}\ f(x) \bullet \cot f(x) \bullet f^{1}(x)$	___
$\sin^{-1} f(x)$	$\frac{f'(x)}{\sqrt{1-[f(x)]^2}}$	___
$\cos^{-1} f(x)$	$\frac{-f'(x)}{\sqrt{1-[f(x)]^2}}$	___
$\tan^{-1} f(x)$	$\frac{f'(x)}{1+[f(x)]^2}$	___
$\cot^{-1} f(x)$	$\frac{-f'(x)}{1+[f(x)]^2}$	___
$\sec^{-1} f(x)$	$\frac{f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$	___
$\mathrm{cosec}^{-1} f(x)$	$\frac{-f'(x)}{f(x)\sqrt{[f(x)]^2-1}}$	___
$\sin^2(ax)$	___	$\frac{x}{2}-\frac{\sin(2ax)}{4a}+C$
$\cos^2(ax)$	___	$\frac{x}{2}+\frac{\sin(2ax)}{4a}+C$
$\tan^2(ax)$	___	$\frac{1}{a}\tan(ax)-x+C$
$\cot^2(ax)$	___	$-\frac{1}{a}\cot(ax)-x+C$

INTEGRATION

$\int f(x)\, g'(x)\, dx = f(x)\, g(x) - \int f'(x)\, g(x)\, dx$

$\int [f(x)]^n \bullet f'(x)dx = \dfrac{[f(x)]^{n+1}}{n+1} + C \qquad (n \neq -1)$

$\int \dfrac{f'(x)}{f(x)} dx = \ln f(x) + C$

APPLICATIONS OF INTEGRATION

AREAS

$A_x = \int_a^b y dx \,;\, A_x = \int_a^b (y_1 - y_2)\, dx$

$A_y = \int_a^b x dy \,;\, A_y = \int_a^b (x_1 - x_2)\, dy$

VOLUMES

$V_x = \pi \int_a^b y^2 dx \,;\, V_x = \pi \int_a^b \left(y_1^2 - y_2^2\right) dx$

$V_y = \pi \int_a^b x^2 dy \,;\, V_y = \pi \int_a^b \left(x_1^2 - x_2^2\right) dy$

SECOND MOMENT OF AREA

$I_x = \int_a^b r^2 dA \,;\, I_y = \int_a^b r^2 dA$

MOMENTS OF INERTIA

Mass = density × volume

$M = \rho V$

DEFINITION: $I = m\, r^2$

GENERAL: $I = \int_a^b r^2 dm = \rho \int_a^b r^2 dV$